PERGAMON

Energy Conversion & Management 40 (1999) 59–69

ENERGY CONVERSION & MANAGEMENT

# Steady state voltage instability assessment in a power system

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Received 23 August 1997

**Abstract**

With the increase of loading and exploitation of power transmission system and also due to the improved optimized operation, the problem of voltage stability and voltage collapse attracts more and more attention. The phenomenon of possible steady state voltage instability is creating serious concern among operators of large interconnected power systems. Maintaining adequate system voltage has become a major problem, as utilities are being forced to operate the system close to the thermal capability or steady state voltage stability limit. A voltage collapse can take place in power systems or subsystems quite abruptly, which requires improved continuous monitoring of the system state. There are different methods used to study the voltage collapse phenomenon, such as the Jacobian method, the simplified Jacobian method, the multiple load flow method and the voltage collapse proximity indicator method. It is proposed to modify the method of multiple load flow to obtain the weakest buses and the maximum injected powers for each bus. The method of voltage collapse proximity indicators is also simplified. These different methods are applied on a simple power system to study the problem of voltage collapse, and a comparison is made between them with indicating the merits and demerits of each. © 1998 Elsevier Science Ltd. All rights reserved.

*Keywords:* Voltage collapse; Voltage instability; Indicator; Eigenvalues; Eigenvectors; Load flows

## 1. Introduction

Stability is one of the most important problems in power system operation and control. Conventional stability has been the ones in real power, such as steady state stability, dynamic stability and transient stability. The voltage instability phenomenon means the ones where receiving end voltages get much lower values than the nominal ones. The phenomenon of

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PII: S0196-8904(98)00095-8

steady state voltage instability in an electric power network is characterized by a progressive decline of voltage, which can occur because of the inability of the network to meet increasing demand for reactive power. The process of voltage instability is generally triggered by some form of disturbance or change in operating conditions which create increased demand for reactive power which is in excess of what the system is capable of supplying [1]. The disturbance or change in system operating conditions, which causes an increase in reactive demand, can be small or large. As transmission systems become more stressed due to the increased loads and large inter-utility power transfers, an efficient system operation is becoming increasingly threatened due to problems of voltage stability and collapse. The term voltage instability is generally used to describe situations in which a disturbance, an increase in loads or other system change, causes bus voltages to vary significantly from their desired operating range in such a way that standard mechanisms of proper intervention or automatic system controls fail to halt this deviation. If bus voltages ultimately fall in a more rapid decline, leading to loss of operations of the network, the term voltage collapse is applied. These voltage related threats to system security are expected to become more severe over the next decade as demand for electric power rises, while economic and environmental concerns limit the construction of new transmission and generation facilities [2].

The study of voltage instability problems has, due to the above collapses and other disturbances caused by voltage stability problems, become an important and interesting area of research and studies. Voltage instability is largely determined by load characteristics and the available means of voltage control [3]. For true voltage instability, at least a part of the total load must be of the self-restoring (constant MVA) type. There are both static and dynamic aspects involved in voltage stability. The purpose of a static voltage stability index is to quantify, in some respect, how 'close' a particular operating point is to the point of steady state voltage collapse and, therefore, to estimate the steady state voltage stability limit for the examined operating point of the studied power system. The information obtained could then be used for setting transfer limits in the network during power system planning studies [4]. In a voltage stability analysis, it is, therefore, imperative that a dynamic system formulation, including the pertinent load dynamics, be employed. For more accurate analysis of dynamic voltage stability, the system model includes excitation systems, under load tap-changers, capacitors and power system stabilizers in addition to network equations. For dynamic voltage stability enhancement, a parameter optimization technique with a model performance measure is used to determine optimal control parameters [5].

Modification in multiple load flow is presented to obtain the weakest buses, the maximum injected powers ($P$ and/or $Q$) and the voltage margin for each bus. Simplification in voltage collapse proximity indicators is also introduced by replacing the submatrices of the bus-admittance matrix by its imaginary parts.

## 2. Methods for predicting voltage instability

Methods for predicting voltage instability in power systems can be categorized either into steady state or dynamic methods. The dynamic methods are very consuming in computation time and the time required to analyse the results, while the static methods with their much less computing time, together with their ability to provide sensitive information and to determine the degree of stability via calculating either a physical margin (load margin, reactive power margin, etc.) or a measure related to the distance to collapse, are being widely used to provide a close observation of the problem.

There are different methods used for solving the voltage collapse problem. Some of these methods are [6]: 1, Testing the Jacobian matrix of the load flow calculation; 2, simplified Jacobian matrix; 3, utilizing multiple load flow solutions to determine a measure of the proximity of the system to voltage collapse; 4, finding voltage collapse proximity indicators. These methods are introduced and applied on a simple 2-generator 5-bus power system with modifications in methods 3 and 4.

## 3. Theoretical formulation

### 3.1. The Jacobian method (JM)

Venikov et al. [7] was the first to relate power system stability to the load flow Jacobian. In this work, it is shown that, with some assumptions ($P$ and $V$ are specified for all generator buses, neglecting damping for all of the generators) and using the Newton-Raphson method in the polar form, Eq. (1), the determinant of the load flow Jacobian becomes equal to the product of the eigenvalues of the system.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} F\theta & F_v \\ G\theta & G_v \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \tag{1}$$

where $\Delta P$, incremental change in bus real power; $\Delta Q$, incremental change in bus reactive power injection; $\Delta\theta$, incremental change in bus voltage angle; $\Delta V$, incremental change in bus voltage magnitude; $F\theta$, first derivative of active power with respect to bus voltage angle; $F_v$, first derivative of active power with respect to voltage magnitude; $G\theta$, first derivative of reactive power with respect to bus voltage angle; $G_V$, first derivative of reactive power with respect to voltage magnitude.

From this equation, when a change takes place in the sign of the determinant, one of the eigenvalues, at least, has crossed the imaginary axis from the stable to the unstable side. In Ref. [8], the minimum singular value of the load flow Jacobian matrix is taken as the voltage collapse proximity indicator, and the sensitivities of this indicator to variation in both load and controls are derived.

### 3.2. The simplified Jacobian method (SJM)

In Refs. [9] and [10], the same indicator, the minimum singular value, is not used for the load flow Jacobian as in Ref. [8] but for a matrix $G_s$ derived from Eq. (1) where as $\Delta P = 0$, $\Delta Q$ will be a function of $\Delta V$ only. The reduced Jacobian matrix $G_s$ becomes

$$G_s = \frac{\Delta Q}{\Delta V} = G_v - G\theta F^{-1}\theta F_v. \tag{2}$$

The work of Ref. [9] has shown that $G_s$ as defined above, is the best submatrix derived from the power flow Jacobian matrix to be used for identifying steady state voltage instability problems. In Ref. [11], the same matrix is used again for the purpose of voltage stability evaluation but, in this case, by eigenvalues and modal analysis. The eigenvalues of the reduced Jacobian, as well as the left and right eigenvectors, are calculated. As long as all the eigenvalues are positive, the system is stable. The left and right eigenvectors are used to determine the critical modal voltages and reactive powers.

### 3.3. The multiple load flow solution (MLF)

Tamura et al. [12] investigate the relationship between voltage instability and multiple load flow solutions in which the multiple load flow solutions problem, which tends to occur in heavy loaded conditions [13], seems to be related to the voltage instability. A closely located multiple solution pair is worthy of attention in the multiple solutions from the stand point of practical power system operations, because both of them seem to be operable for their close location. They seem to be related to voltage instability for the next reasons: the Jacobian rank of the load flow calculations using the Newton–Raphson method reduces [14], and the load flow sensitivity for a multiple load flow solution pair becomes opposite to each other. In this investigation, the relationship between voltage instability and multiple load flow solutions assumed that one is stable and the other is unstable, if there is a pair of multiple load flow solutions.

The conditions given in this analysis are given as: (1) active power $P$ and voltage magnitude $|V|$ are specified for each of the generator buses, (2) voltages are represented by the polar coordinates and (3) damping of the generators is neglected.

Linearized equations for $M$ generators in electric power system conditions, say "$S_0$" are given by Eq. (3)

$$\left[\dot{Z}\right]_{2M\times 1} = \left[A(S)\right]_{2M\times 2M} [Z]_{2M\times 1} + [\delta]_{2M\times 1} \tag{3}$$

where $[Z]$, state vector (phase angle and angular frequency of generators); $[A(S_0]$, system's Jacobian in conditions $S_0$; $[\delta]$, error vector. Then, the matrix $A$ contains the Jacobian $J$ of the load flow calculation and takes the following form

$$[A] = \begin{bmatrix} 0 & I \\ CJ & 0 \end{bmatrix} \tag{4}$$

where $I$, unit matrix, $M \times M$; $C$, diag $(-f/H_1, -f/H_2, \ldots, -f/H_M)$, $M \times M$; $f$, system frequency; $H_j$, inertia constant of the $j$-th machine ($j = 1, 2, \ldots, M$). Besides, the relationship between the eigenvalues $\lambda_{Ai}$ ($i = 1, 2, \ldots, 2M$) of the matrix and its determinant $|A|$ is given by Eq. (5)

$$|A| = \sum_{i=1}^{2M} \lambda_{Ai}. \tag{5}$$

Representing determinant $|J|$ by $\lambda_{Ai}$, we get Eq. (6)

$$|J| = \sum_{j=1}^{M} \frac{Hj}{\pi f} \sum_{i=1}^{2M} \lambda_{Ai}. \tag{6}$$

Using Eq. (6), the signs of $\lambda_{Ai}$ can be changed by means of the sign of $|J|$. Then, we confirm that all the real parts of the eigenvalues, $\mathrm{Re}(\lambda_{Ai})$, are negative in the conditions $S_0$ and, furthermore, that the sign of $[A(S_0)]$ is the same as the one of $|J|$. After that, the load flow calculation is performed each time the operating point changes from $S_0$ to $S_1, S_2, \ldots, S_K$ in response to a transition of an electric power system as denoted in Fig. 1

At those points, system stability is checked by means of Eq. (7).

$$F(S_K)\left[\begin{array}{ll} = F(S_0) & \text{stable} \\ \neq F(S_0) & \text{unstable} \end{array}\right. \tag{7}$$

where

$$F(S_K) = \mathrm{sign}\ \{|J(S_K)|\}.$$

This method shows that only the power system is stable or unstable according to the sign of the Jacobian determinant $|J|$. Therefore, it is intended to modify this method by performing the load flow after changing the active and/or reactive powers by some amount at each bus and each time, the voltage at each bus is calculated until it reaches a certain value which will be the critical value, and otherwise, the system will be unstable. From this modification, the weakest buses, the voltage profile and the maximum injected power at each bus can be obtained.

### *3.4. The voltage collapse proximity indicators (VCPI)*

Different indicators have been proposed to assess the proximity of the system to voltage collapse. Kessel et al. [15] suggests a method for the on line testing of a power system, which is aimed at the detection of voltage instabilities. Thereby, an indicator $L$ varies in the range between 0 (no load of system) and 1 (voltage collapse). Values close to one indicate proximity to power flow divergence. Based on the basic concept of such an indicator, various models are derived which allow predicting a voltage instability or the proximity of a collapse under various contingencies, such as loss of generators or lines as well as load variations. The indicator uses information of a normal load flow. The advantage of the method lies in the simplicity of the numerical calculation and the expressiveness of the results. In this approach, a

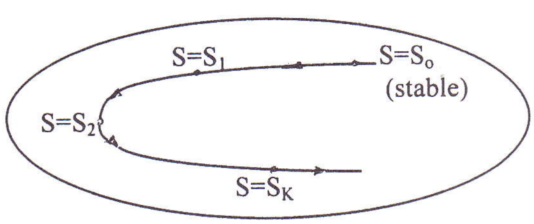


Fig. 1. Transition of system state and stability.

local indicator $L_j$ can be determined for each node $j$ by

$$L_j = \left| S_j^+ / \left( Y_{jj}^{+*} \cdot V_j^2 \right) \right| \tag{8}$$

where $Y_{jj}^+$, transformed admittance = $(1/Z_{jj})$; $V_j$, consumer node voltage; $S_j^+$, transformed power = $S_j + S_j^{\mathrm{cor}}$; and $S_j^{\mathrm{cor}}$ is given by

$$S_j^{\mathrm{cor}} = \left[ \sum_{i\varepsilon\alpha} \left( Z_{ji} * / Z_{jj} * \right) \cdot (S_i/V_i) \right] \cdot V_j \tag{9}$$

and $\alpha_L$ is the set of consumer nodes. Therefore the nodal voltage $V_j$ is affected by the nodal power $S_j$ and an equivalent power $S_j^{\mathrm{cor}}$ which stems from the other loads of the system.

For stable situations, the condition $L_j \leq 1$ must not be violated for any of the nodes $j$. Hence, a global indicator $L$ describing the stability of the complete subsystem is given by Eq. (10).

$$L = \max_{j \in \alpha_L} \left( L_j \right). \tag{10}$$

One way of determining $L_j$ is given by

$$L_j = |L_j| = |1 - \frac{\sum\limits_{i \in \alpha_G} C_{ji} V_j}{V_j}| \qquad j \in \alpha_L \tag{11}$$

where $\alpha_L$, set of load buses; $\alpha_G$, set of generator buses; $V_j$, complex voltage at load bus $j$; $V_I$, complex voltage at generator bus $I$; $C_{ji}$, elements of matrix $C$ determined by

$$[C] = -[Y_{LL}]^{-1}[Y_{LG}] \tag{12}$$

where $[Y_{LL}]$ and $[Y_{LG}]$ are submatrices of the $Y$-bus matrix.

To reduce the time and burden of calculations, it is proposed to take the imaginary parts of the submatrices of the bus-admittance matrix instead of these submatrices, as given in Eq. (13).

$$[C] = -[B_{LL}]^{-1}[B_{LG}] \tag{13}$$

where $[B_{LL}]$ and $[B_{LG}]$ are the imaginary parts of the matrices $[Y_{LL}]$ and $[Y_{LG}]$, respectively. The computation speed of these indicators is very fast, and it is easy to modify a load flow program in order to obtain these indicators. These indicators can be used for on-line monitoring of a power system. With the help of these indicators, critical load buses can be identified. Thus, the important outcome of the presented theory is $L < 1$ for stability to be guaranteed.

This theory is exact when two conditions are fulfilled: 1, All generator voltages remain unchanged, amplitude and phase wise; 2, the nodal currents respond directly proportional to the current $I_j$ and indirectly proportional to the voltage $V_j$ at the node $j$ under consideration. The drawback of this method is that it fails to consider the operating constraints of system equipment, such as the VAR limits of the generators. This is an important consideration because, when a generator reaches its VAR limit, the terminal voltage can no longer be

Table 1
Impedances and line charging

| From bus | To bus | Impedance | Line charging |
|---|---|---|---|
| 1 | 2 | $0.02 + j0.06$ | $0.0 + j0.03$ |
| 1 | 3 | 0.<br>$08 + j0.24$ | $0.0 + j0.025$ |
| 2 | 3 | $0.06 + j0.18$ | $0.0 + j0.02$ |
| 2 | 4 | $0.06 + j0.18$ | $0.0 + j0.02$ |
| 2 | 5 | $0.04 + j0.12$ | $0.0 + j0.15$ |
| 3 | 4 | $0.01 + j0.03$ | $0.0 + j0.01$ |
| 4 | 5 | $0.08 + j0.24$ | $0.0 + j0.025$ |

controlled. Under this condition, the machine model has to be modified, resulting in a change in the system performance pattern.

## 4. Applications and results

Fig. 2 shows the sample power system [16]. The transmission line impedances and line charging admittances are given in Table 1. The scheduled generation and loads and the assumed per unit bus voltages are given in Table 2.

### 4.1. Jacobian method results

The eigen-values and eigen-vectors of the Jacobian matrix are tabulated in Table 3.

It is obvious that all the eigen-values are positive which means that the system is stable, but as bus 5 has the lowest eigen-value (3.55, 3.96), we can say that it will be the weakest bus.

### 4.2. Simplified Jacobian method results

Table 4 gives also the eigenvalues and eigenvectors of the simplified matrix $G_s$. It is clear that its results almost agree with those obtained by the first method.

Table 2
Scheduled generation and loads and assumed bus voltages

| | | Generation | | Load | |
|---|---|---|---|---|---|
| Bus No. | Assumed bus voltage | MW | MVAR | MW | MVAR |
| 1 | $1.06 + j0.0$ | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | $1.00 + j0.0$ | 40 | 30 | 20 | 10 |
| 3 | $1.00 + j0.0$ | 0.0 | 0.0 | 45 | 15 |
| 4 | $1.00 + j0.0$ | 0.0 | 0.0 | 40 | 5 |
| 5 | $1.00 + j0.0$ | 0.0 | 0.0 | 60 | 10 |

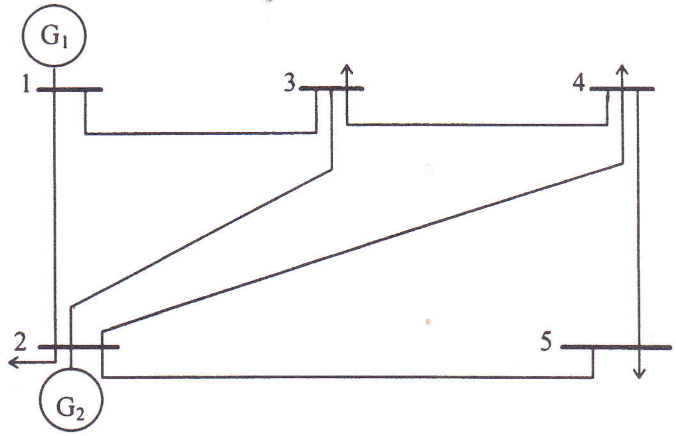


Fig. 2. One line diagram of the sample system.

### 4.3. Multiple load flow solution results

#### 4.3.1. Without modification

Table 5 shows the signs of the Jacobian determinant $|J|$ at each system transition ($S_K$).

Table 3
Eigen-values and eigen-vectors of the jacobian matrix

| Bus No. | | Eigen-values | Eigen-vectors ΔP | | | | ΔQ | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | | 80.78 | 0.014 | 1.00 | 0.738 | 0.310 | 0.746 | 0.038 | 0.047 | −0.080 |
| 3 | | 43.12 | −0.99 | −0.05 | −0.004 | 0.209 | −0.62 | −0.691 | −0.054 | −0.270 |
| 4 | ΔP | 11.31 | 1.00 | −0.08 | −0.041 | 0.222 | −0.728 | −0.789 | −0.052 | −0.283 |
| 5 | | 3.55 | −0.06 | −0.10 | 0.016 | 0.416 | −0.595 | −0.363 | −0.01 | −0.256 |
| 2 | | 69.94 | 0.00 | −0.38 | −0.886 | −0.03 | 1.00 | 1.00 | 0.591 | 0.590 |
| 3 | | 38.28 | −0.02 | 0.06 | 0.567 | 0.575 | −0.139 | −0.24 | 0.855 | 0.851 |
| 4 | ΔQ | 16.65 | 0.02 | 0.05 | 0.671 | 0.684 | −0.085 | −0.224 | 0.911 | 0.908 |
| 5 | | 3.96 | 0.0 | 0.10 | 1.00 | 1.00 | 0.819 | 0.313 | 1.00 | 1.00 |

Table 4
Eigen-values and eigen-vecotrs of the matrix $G_s$

| Bus No. | Eigen-values | Eigen-vectors | | | |
|---|---|---|---|---|---|
| 2 | 78.1909 | 0.01450 | 1.0000 | 0.07909 | 0.5537 |
| 3 | 42.0495 | −0.99513 | −0.14555 | −0.61526 | 0.8650 |
| 4 | 14.2083 | 1.00000 | −0.17753 | −0.54886 | 0.92224 |
| 5 | 3.8131 | −0.06685 | −0.26796 | 1.0000 | 1.0000 |

Table 5
Multiple load flow without modification

| | System transition ($S_K$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $S_0$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ | $S_8$ | $S_9$ | $S_{10}$ |
| Sign of $\|J\|$ | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve |
| The system is stable | | | | | | | | | | | |

### 4.3.2. *With modification*

This modification gives in addition to the results given in Table 5, the voltages, active and reactive powers for each bus as shown in Table 6.

It is obvious that bus 5 is the weakest one because its voltage is rapidly reduced compared to other buses as $P$ and/or $Q$ are changed. These results agree with the other methods.

## 4.4. *Voltage collapse proximity indicators results*

The indicators for different load buses without simplification by taking $[Y_{LL}]$, $[Y_{LG}]$ and with simplification by taking $[B_{LL}]$, $[B_{LG}]$ are tabulated in Table 7.

As all the indicators ($L_j$) are less than one in both cases, the system is stable. The weakest buses can be arranged as 5, 4 and 3, respectively. Bus 5 has the largest indicator value, then bus 4 and finally bus 3. The comparison between the applied methods is tabulated in Table 8.

Table 6
Multiple load flow solutions

| | System transition ($S_K$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Var. | $S_0$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ | $S_8$ | $S_9$ | $S_{10}$ |
| Sign of $\|J\|$ | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve | + ve |
| $P_2$ | 0.2 | −0.2 | −0.8 | −2.0 | 0.2 | −0.2 | −0.2 | −0.2 | 0.2 | 0.2 | 0.2 |
| $Q_2$ | 0.2 | −0.2 | −0.8 | −2.0 | 0.2 | −0.2 | −0.2 | −0.2 | 0.2 | 0.2 | 0.2 |
| $V_2$ | 1.06 | 1.04 | 0.99 | 0.88 | 1.04 | 0.97 | 0.98 | 0.9 | 0.89 | 0.87 | 0.86 |
| $P_3$ | −0.45 | −0.45 | −0.45 | −0.45 | −0.8 | −1.0 | −1.0 | −1.0 | −1.0 | −1.0 | −1.0 |
| $Q_3$ | −0.15 | −0.15 | −0.15 | −0.15 | −0.5 | −1.0 | −1.0 | −1.0 | −1.0 | −1.0 | −1.0 |
| $V_3$ | 1.05 | 1.03 | 0.99 | 0.92 | 1.03 | 0.88 | 0.89 | 0.83 | 0.76 | 0.73 | 0.71 |
| $P_4$ | −0.4 | −0.4 | −0.4 | −0.4 | −0.4 | −0.8 | −0.8 | −0.8 | −0.8 | −0.8 | −0.8 |
| $Q_4$ | −0.05 | −0.05 | −0.05 | −0.05 | −0.05 | −0.02 | −0.02 | −0.02 | −0.8 | −0.8 | −0.8 |
| $V_4$ | 1.05 | 1.02 | 0.99 | 0.91 | 1.01 | 0.89 | 0.8 | 0.83 | 0.74 | 0.72 | 0.7 |
| $P_5$ | −0.6 | −0.6 | −0.6 | −0.6 | −0.6 | −0.6 | −0.2 | −0.8 | −0.8 | −1.0 | −1.2 |
| $Q_5$ | −0.1 | −0.1 | −0.1 | −0.1 | −0.1 | −0.1 | −0.2 | −0.6 | −0.8 | −1.0 | −1.4 |
| $V_5$ | 1.04 | 1.01 | 0.97 | 0.88 | 1.01 | 0.9 | 0.93 | 0.8 | 0.7 | 0.62 | 0.56 |

Table 7
Voltage proximity indicators

| | Indicator ($L_i$) | |
|---|---|---|
| Load bus | Without simplification | With simplification |
| 3 | 0.0585 | 0.05937 |
| 4 | 0.0611 | 0.06272 |
| 5 | 0.0700 | 0.07067 |

## 5. Conclusion

Voltage stability assessment is the main concern of this paper. Encouraging results are obtained when applying the methods on a simple system to study the voltage collapse phenomenon taking in consideration the suggested modification and simplification. Using the method of testing the Jacobian matrix of the load flow calculation, the results illustrate that this method defines the weakest buses arrangement but needs heavy computation burden and does not differentiate between steady state stability problems caused by voltage instability or

Table 8
Comparison of the applied methods

| Method | Weakest bus | Advantages | Disadvantages |
|---|---|---|---|
| JM | 5 | Give accurate results. The weakest buses can be arranged | Needs heavy computation burden. Not differentiate between voltage or angle instability |
| SJM | 5 | Fast, simple, less computing time, and reasonable results. Voltage instability depends only on injected reactive power | Voltage margin for each bus not determined |
| MLF without modification | 5 | Fast and simple | Not indicate the maximum injected power and voltage at each bus |
| MLF with modification | 5 | Give information about system state such as voltages and injected powers at each bus | Large computing time |
| VPI without simplification | 5 | Simple in the numerical calculation | Fail to consider the operating constraints of the system equipment such as the VAR limits of the generators. |
| VPI with simplification | 5 | More simple in numerical calculations and reasonable results. Used for on-line applications | Not identify the voltage margin and maximum injected power at each bus |

due to other reasons, such as angle instability. The simplified Jacobian is fast, simple and needs less computing time because the voltage instability depends only on the reactive power. The suggested modification for the multiple load flow method provides detailed information for the system state (voltages and injected powers at each bus). The proposed simplification for the voltage proximity indicator method reduces the burden and time of calculations because, the matrix inversion is made for the imaginary part only of the admittance submatrices instead of a complex matrix inversion of these submatrices. Also, the results obtained from the proposed method are very close and agree with the results obtained from the voltage proximity indicator method without simplification.

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